Title: Cherednik operators and radial parts of non-symmetric elements

Abstract: Let G be a non-compact real semisimple Lie group, $G = KAN$ its Iwasawa decomposition, and $W = W(A, G)$ the Weyl group. A *hypergeometric function* associated to the root system (Heckman-Opdam's hypergeometric function) is a W-invariant solution for a certain system of W-invariant differential-difference equations constructed via Cherednik operators. It equals the restriction of a spherical function on G/K when the multiplicity parameter corresponds to G. On the other hand, the solution space for the above equation system contains many non-symmetric elements such as *non-symmetric hypergeometric functions* [2]. In this talk we exhibit a direct connection of general non-symmetric solutions with the harmonic analysis of G/K by the following result:

Let $U(\mathfrak{g})$ be the universal enveloping algebra of the complexified Lie algebra $\mathfrak{g}$ of G. For any $D \in U(\mathfrak{g})$ let $\gamma(D)$ denote its image under the Harish-Chandra homomorphism and $T(\gamma(D))$ the corresponding Cherednik operator. It is well-known that for $D \in U(\mathfrak{g})^K$ and $f \in C^\infty(G/K)^K$

$$(Df)|_A = T(\gamma(D))(f|_A). \tag{$\star$}$$

This formula holds for more general combinations of $D \in U(\mathfrak{g})$ and $f \in C^\infty(G/K)$. Let S be the set of *single-petaled* K-types (cf. [1]) and put

$$\alpha = \sum_{\sigma \in S} \deg \sigma (\text{character of } \sigma) \in C^\infty(K).$$

Then ($\star$) holds for (D, f) such that $\mathrm{Ad}_{U(\mathfrak{g})}(\alpha) D = D$ and $f \in C^\infty(G/K)^K$, and in addition, for (D, f) such that $D \in U(\mathfrak{g})^K$ and $\alpha *_K f = f$.

A similar result also holds for the Cartan motion group and rational Dunkl operators.

[1] H. Oda, *Generalization of Harish-Chandra's basic theorem for Riemannian symmetric spaces of non-compact type*, Adv. Math., **208** (2007), 549–596

[2] E. M. Opdam, *Harmonic analysis for certain representations of graded Hecke algebras*, Acta Math., **175** (1995), 75–121.