List of main talks

Apr 2002	*Floer homology and Dehn twists* Ialk at Tel Aviv University
March 2003	*Twisted cotangent bundles and periodic orbits* Talk held at ETH Zürich
June 2003	*Twisted cotangent bundles and periodic orbits* Talk held at Hokkaido University
Sep 2003	*Twisted cotangent bundles and periodic orbits* Talk at Okayama University
Nov 2003	*The Arnold-Givental conjecture and moment Floer homology* Talk at Tokyo University
Dec 2003	*The Arnold-Givental conjecture and moment Floer homology* Talk at Kyoto University
May 2004	*The symplectic vortex equations* Lecture series held at Hokkaido University
Oct 2004	*Finite dimensional approximations for the symplectic vortex equations* Talk at Tokyo University
Nov 2004	*Finite dimensional approximations for the symplectic vortex equations* Talk at KIAS (Korean Institute for Advanced Sciences)
Nov 2004	*The Symplectic vortex equations* Talk at KIAS (Korean Institute for Advanced Sciences)
Dec 2004	*Morse functions with Lagrange multipliers* Talk at Tokyo University
Feb 2005	*Morse functions with Lagrange multipliers* Talk at "Hiroshima geometry meeting"

Publications

U.Frauenfelder, *The Arnold-Givental conjecture and moment Floer homology*, Int.Math.Rex.Not. 2004, no 42, 2179-2269

U.Frauenfelder, F.Schlenk *Volume growth in the component of the Dehn-Seidel twist*, to appear in GAFA.

U.Frauenfelder, F.Schlenk, *Hamiltonian dynamics on convex symplectic manifolds*, math.SG/0303282
submitted for publication.

U.Frauenfelder, V.Ginzburg, F.Schlenk *Energy capacity inequalities via an action selector*, math.DG/040204.

U.Frauenfelder, *Finite dimensional approximations for the symplectic vortex equations*, math.SG/0408102

Research I carried out so far

The Arnold-Givental conjecture and moment Floer homology

The question about fixed points of symplectic mappings is an old problem of celestial mechanics. The Arnold conjecture in its homological form states that the number of fixed points of a nondegenerate Hamiltonian symplectomorphism of a compact symplectic manifold is bounded from below by the sum of the Betti numbers of the symplectic manifold. After the pioneering works of Eliashberg [?] and Conley and Zehnder [?] the breakthrough came in the late 1980s with Floer's discovery of his semi-infinite dimensional Morse homology on the free loop space called Floer homology. Floer's proof of the Arnold conjecture for monotone symplectic manifolds in [?] was later pushed through by Hofer-Salamon [?] and Ono [?] to the semipositive case and by Fukaya-Ono [?] and Liu-Tian [?] to the general case.

The Arnold-Givental conjecture is a natural generalization of the Arnold conjecture. Its precise formulation is as follows. Assume that (M, ω) is a $2n$-dimensional compact symplectic manifold and $L \subset M$ is a compact Lagrangian submanifold of M, which is the fixed point set of an antisymplectic involution, i.e. there exists $R \in \mathrm{Diff}(M)$ such that

$$R^*\omega = -\omega, \quad R^2 = \mathrm{id}, \quad \mathrm{Fix}(R) = L.$$

Suppose further that ϕ is a Hamiltonian isotopy of M, i.e. there exists a smooth family $H_t \in C^\infty(M)$, $t \in [0,1]$, of Hamiltonian functions on M, such that ϕ is the time-1-map of the time dependent Hamiltonian vector field of H_t. The Arnold-Givental conjecture reads.

Conjecture (Arnold-Givental): *Assume that L and $\phi(L)$ intersect transversally. Then the number of intersection points of L and $\phi(L)$ can be estimated from below by the $\mathbb{Z}_2$-Betti numbers of L, i.e.*

$$\#(L \cap \phi(L)) \geq \sum_{k=0}^{n} b_k(L; \mathbb{Z}_2).$$

The Arnold conjecture - at least for $\mathbb{Z}_2$-coefficients - follows immediately from the Arnold-Givental conjecture by considering the diagonal in the symplectic manifold $(M \times M, \omega \times -\omega)$ which is a Lagrangian submanifold fixed under the antisymplectic involution given by interchanging the two factors. However, the Arnold-Givental conjecture is much more general than the Arnold conjecture and is still open in its full generality up to now.

In my thesis [?] I introduced a class $\mathcal{L}$ of Lagrangian submanifolds in Marsden-Weinstein quotients, which are fixed point sets of an antisymplectic involution. I proved the following theorem in [?].

Theorem: *Assume that L belongs to $\mathcal{L}$. Then the Arnold-Givental conjecture holds true for L.*

I point out that the Lagrangians which belong to $\mathcal{L}$ are in general neither monotone nor semipositive so that Theorem A cannot be deduced from results about the Arnold-Givental conjecture established so far, see [?, ?, ?, ?].

In order to prove Theorem A one computes moment Floer homology, which was introduced in my thesis [?]. The main advantage of moment Floer homology compared to the original Floer homology are better compactness properties. Assume that there is a Hamiltonian group action on a symplectic manifold. Then the Moment action functional is defined to be Floer's original action functional together with a Lagrange multiplier to the constraint given by the zero set of the moment map. The Lagrangian constraint guarantees that the critical orbits of the Moment action functional lie on the zero set of the moment map so that by quotienting out the gauge action they can be identified with orbits in the Marsden-Weinstein quotient. It turns out that the flow lines of the Moment action functional are solutions of the symplectic vortex equations. These equations were discovered independently by D.Salamon and I.Mundet (see [?], [?], and [?]) and special cases of them are known in the physics literature as gauged sigma models. There are many cases for which one can achieve compactness for the symplectic vortex equations while the compactness for the Floer equations fails due to the bubbling phenomenon in the Marsden-Weinstein quotient.

In [?] moment Floer homology is computed. The computation makes use of a symmetry which comes from the antisymplectic involution. However, this symmetry cannot in general be applied directly, since the almost complex structure which one needs to define the boundary operator in moment Floer homology has to be perturbed to achieve transversality and this perturbation cannot be chosen in general to be invariant under the symmetry. To overcome this problem I use in [?] the abstract perturbation theory of Fukaya and Ono [?] and show that the abstract perturbation can be chosen invariant under the symmetry.

Hamiltonian dynamics

In a joint work with Felix Schlenk we study the dynamics of Hamiltonian diffeomorphisms on convex symplectic manifolds. Our main tool is a spectral norm. Spectral norms were considered previously by Viterbo, Schwarz and Oh, see [?, ?, ?, ?]. The new point is that the symplectic manifold for which we define the spectral norm is allowed to have contact boundary. In order to define a spectral norm in this setting we establish the Piunikhin-Salamon-Schwarz isomorphism [?] for symplectic manifolds with contact boundary. Our main result is a new existence theorem for closed orbits of a charge in a magnetic field on almost all small energy levels. More precisely, we prove the following theorem.

Theorem: *Let (N, g) be a closed Riemannian manifold and let α be a one-form on N such that $d\alpha$ does not vanish identically. Then there exists $\epsilon > 0$ and a subset $\Sigma \subset (0, \epsilon)$ of full Lebesgue measure, such that for every $\delta \in \Sigma$ there exists a closed orbit $x \in C^\infty(S^1, E_\delta)$ on $E_\delta := \{(q,p) \in T^*N : \frac{1}{2}|p|^2 = \delta\}$ which is contractible in T^*N and satisfies*

$$\dot{x} = X_H(x)$$

where the Hamiltonian function $H\colon T^*N \to \mathbb{R}$ is given by

$$H(p, q) = \frac{1}{2}|p|^2$$

and X_H is the Hamiltonian vector field of H with respect to the twisted symplectic complex structure

$$\omega_\alpha = -d\lambda - d\pi^*\alpha$$

on T^*N. Here λ denoted the canonical one-form on the cotangent bundle and $\pi\colon T^*N \to N$ the canonical projection.

The results of [**?**] were generalized in [**?**] were we introduce the concept of an action selector in an axiomatic way. We show how the existence of an action selector leads to sharp energy inequalies between the Gromov width, the Hofer-Zehnder capacity, and the displacement energy. We also obtain sharp lower bounds for the smallest action of a closed characteristic on contact type hypersurfaces.

Slow entropy and symplectomorphism of cotangent bundles

In a joint work with Felix Schlenk [**?**] we compute a class of Lagrangian Floer homologies where the first Lagrangian submanifold is a fiber of the cotangent bundle of a CROSS and the second Lagrangian submanifold is the n'th iterate of the Dehn-Seidel twist applied to another fiber. It turns out that the rank of the Floer homology is proportional to n. This solves in a more general context a conjecture stated by Polterovich in [**?**]. In [**?**] we interpret this result in terms of an entropy-type invariant which measures the polynomial volume growth of submanifolds under the iterates of a map.

Finite dimensional approximations for the symplectic vortex equations

In [**?**], Cohen, Jones, and Segal conjectured the existence of a pro-spectrum whose homology should coincide with Floer homology and which can be used to define Floer homotopy type, Floer K-theory as well as Floer complex cobordism. Kronheimer and Manolescu constructed this pro-spectrum in [**?**, **?**] for Seiberg-Witten Floer homology using Furuta's idea of finite dimensional approximation, see [**?**], and the theory of the Conley index, see [**?**, **?**].

The symplectic vortex equations were introduced in [**?**, **?**, **?**]. They were used in [**?**, **?**] to define moment Floer homology. In [**?**] I construct the Conley indices for moment Floer homology of symplectic toric orbifolds following the approach taken by Kronheimer and Manolescu and compute them. It turns out that the Conley indices are isomorphic to the Thom space of the normal bundle of toric map space which was introduced by Givental [**?**, **?**] as a model for Floer homology of symplectic toric orbifolds, see also [**?**, **?**].

References

[BCK] A. Bertram, I. Ciocan-Fontanine, B. Kim, *Gromov-Witten invariants for Abelian and Nonabelian Quotients*, math.AG/0407254.

[CGMS] K. Cieliebak, R. Gaio, I. Mundet, D. Salamon,*The symplectic vortex equations and invariants of Hamiltonian group actions*, Preprint 2001.

[CGS] K. Cieliebak, R. Gaio, D. Salamon, *J-holomorphic curves, moment maps, and invariants of Hamiltonian group actions*, IMRN **10** (2000),813-882.

[CJS] R. Cohen, J. Jones, G. Segal, *Floer's infinite-dimensional Morse theory and homotopy theory*, in the Floer memorial volume, 297-325, Birkhäuser, Basel, 1995.

[C] C. Conley, *Isolated invariant sets and the Morse index*, Amer. Math. Soc., Providence, 1978.

[CZ] C. Conley, E. Zehnder, *The Birkhoff-Lewis fixed point theorem and a conjecture of V.I.Arnold*, Inventiones Mathematicae, **73**, 33-49.

[E] Y. Eliashberg, *Estimates on the number of fixed points of area preserving transformations*, Syktyvkar University preprint, 1979.

[Fl] A. Floer, *Symplectic fixed points and holomorphic spheres*, Communications in Mathematical Physics, **120** (1989), 575-611.

[F1] U. Frauenfelder, *Floer homology of symplectic quotients and the Arnold-Givental conjecture*, thesis, (2003).

[F2] U. Frauenfelder, *The Arnold-Givental conjecture and moment Floer homology*, math.SG/0309373.

[F3] U. Frauenfelder {Finite dimensional approximations for the symplectic vortex equations}, preprint, 2004.

[FGS] U. Frauenfelder, V. Ginzburg, F. Schlenk, *Energy capacity inequalities via an action selector*,math.DG/040204.

[FS1] U. Frauenfelder, F. Schlenk, *Hamiltonian dynamics on convex symplectic manifolds*,math.SG/0303282.

[FS2] U. Frauenfelder, F. Schlenk, *Slow entropy and symplectomorphisms of cotangent bundles*, Preprint.

[FO] K. Fukaya, K. Ono, *Arnold conjecture and Gromov-Witten invariants for general symplectic manifolds*, Topology **38** (1999), 933-1048.

[FOOO] K. Fukaya, Y. Oh, H. Ohto, K. Ono *Lagrangian Intersection Floer theory - Anomaly and Obstruction -*, Kyoto University, 2000.

[F] M. Furuta, *Monopole equation and the 11/8-conjecture*, Math. Res. Lett, **8** (2001), 279-291.

[G-P] O. Garcia-Prada, *A direct existence proof for the vortex equations over a compact Riemann surface*, Bull.London Math.Soc. **26** (1994), 88-96.

[Gi1] A. Givental, *Periodic maps in symplectic topology*, Funct. Anal. Appl. **23:4** (1989), 37-52.

[Gi2] A. Givental, *A symplectic fixed point theorem for toric manifolds*, in [**?**], 445-481.

[Gi3] A. Givental, *Homological Geometry. I. Projective Hypersurfaces*, Selecta Math. (N.S.) 1 (1995), no.2, 325-345.

[Gi4] A. Givental, *A Mirror Theorem for Toric Complete Intersections.* Topological field theory, primitive forms and related topics (Kyoto,1996), pp.141-175, Progr. Math., **160**, Birkhäuser Boston, Boston, MA, 1998.

[Gu] M. Guest, *Quantum cohomology via D-modules*, math.DG/0206212.

[Flm] H. Hofer, C. Taubes, A. Weinstein, and E. Zehnder, eds., *The Floer Memorial Volume,* Birkhäuser 1995.

[HS] H. Hofer, D. Salamon *Floer homology and Novikov rings*, in [**?**], 483-524.

[I] H. Iritani, *Quantum D-module and equivariant Floer theory for free loop spaces*, preprint, 2004.

[KM] P. Kronheimer, C. Manolescu, *Periodic Floer Pro-Spectra from the Seiberg-Witten equations*, preprint, math.GT/0203243.

[La] L. Lazzarini, *Pseudoholomorphic curves and the Arnold-Givental conjecture*, Thesis 1999.

[LT] G. Liu, G. Tian, *Floer homology and Arnold conjecture*, J. Diff. Geom., **49** (1998), 1-74.

[M] C. Manolescu, *Seiberg-Witten-Floer stable homotopy type of three-manifolds with* $b_1 = 0$, preprint, math.DG/0104024.

[Mu] I. Mundet, *Yang-Mills-Higgs theory for symplectic fibrations*, PhD thesis, Madrid, April 1999.

[Oh1] Y. Oh, *Floer cohomology of Lagrangian intersections and pseudoholomorphic disks III. Arnold-Givental conjecture*, in [**?**].

[Oh2] Y. Oh, *Chain level Floer theory and Hofer's geometry of the Hamiltonian diffeomorphism group*, math.SG/0104243.

[Oh3] Y. Oh, *Mini-max theory, spectral invariants and geometry of the Hamiltonian diffeomorphism group*, math.SG/0207214.

[O] K. Ono, *The Arnold conjecture for weakly monotone symplectic manifolds*, Invent. Math. **119** (1995), 519-537.

[OS] P. Oszvath, Z. Szabo, *Holomorphic disks and topological invariants for closed three-manifolds*, math.SG/0101206.

[PSS] S. Piunikhin, D. Salamon, M. Schwarz, *Symplectic Floer-Donaldson theory and quantum cohomology*, Contact and Symplectic Geometry (Cambridge, 1994) (C. B. Thomas, ed.), Publ.Newton Inst., vol. 8, Cambridge University Press, Cambridge, 1996, 171-200.

[P] L. Polterovich, *Slow symplectic maps, continued fractions, and related stories,* Symplectic and contact topology: interactions and perspectives (Toronto, ON/Montreal, QC, 2001), 165-173, Fields Inst. Commun. **35**, Amer. Math. Soc., Providence, RI, 2003.

[Sa] D. Salamon, *Connected simple systems and the Conley index of isolated invariant sets,* Trans. Amer. Math. Soc. **291** (1985), no.1, 1-41.

[Sch1] M. Schwarz, *A quantum cup length estimate for symplectic fixed points*, Invent. Math. **133**,(1998), no.2, 353-397.

[Sch2] M. Schwarz, *On the action spectrum for closed symplectically aspherical manifolds*, Pacific J. Math. **193** (2000), 419-461.

[Vi] C. Viterbo, *Symplectic topology as the geometry of generating functions*, Math. Ann. **292** (1992), 685-710.

[Vl] Y. Vlassopoulos, *Quantum Cohomology and Morse Theory on the Loop Space of Toric Varieties*, preprint, math.AG/0203083.