Research I carried out so far

Moment Floer homology and the Arnold-Givental conjecture

In my thesis (see [Fr1]) I defined an equivariant version of Floer homology, called moment Floer homology, and computed it under some monotonicity assumption. This monotonicity assumption was removed in [Fr2]. Its computation leads to a proof of the Arnold-Givental conjecture for a class of Lagrangian submanifolds in Marsden-Weinstein quotients which are fixpoint sets of an antisymplectic involution, i.e. the number of intersection points of such a Lagrangian with its transversal image under a Hamiltonian isotopy can be estimated from below by the sum of the $\mathbb{Z}_2$-Betti numbers of the Lagrangian. These Lagrangians are in general not semipositiv so that the Arnold-Givental conjecture cannot be proven by other means established so far.

The main motivation to introduce moment Floer homology was to overcome the problem of bubbling. The bubbling phenomenon prevents the relevant moduli spaces from being compact, so that Floer homology cannot be defined in general. Under some topological assumptions on the manifold the bubbling phenomenon can be excluded. However, these assumptions are in general quite restrictiv. There are many examples of Marsden-Weinstein quotients which do not satisfy these assumptions but whose ambient manifolds meet them. Therefore moment Floer homology considers the L^2-gradient flow lines of an action functional on the ambient manifold which consists of Floer's action functional together with a Lagrangian constraint which makes sure that its critical point lie on the zero set of the moment map.

The computation of moment Floer homology makes use of the antisymplectic involution. The idea is to use this involution to show that flow lines appear in pairs and so its effect cancels as long as one considers the homology with $\mathbb{Z}_2$-coefficients. However, this can only be achieved under some nongeneric assumptions on the almost complex structure so that transversality will fail in general as long as one considers just perturbations of the almost complex structure. To make the idea described above precise one needs to introduce abstract perturbations in the sense of Fukaya and Ono. This was carried out in [Fr2] where it was shown how the abstract perturbation has to be carried out to make use of the antisymplectic involution.

Hamiltonian dynamics

In a joint work with Felix Schlenk we study the dynamics of Hamiltonian diffeomorphisms on convex symplectic manifolds. Our main theorem is the following.

Theorem Let (N, g) be a closed Riemannian manifold and let α be a one-form on N such that $d\alpha$ does not vanish identically. Then there exists $\epsilon > 0$ and a subset $\Sigma \subset (0, \epsilon)$ of full Lebesgue measure, such that for every $\delta \in \Sigma$ there exists a closed orbit $x \in C^\infty(S^1, E_\delta)$ on $E_\delta := \{(q, p) \in T^*N : \frac{1}{2}|p|^2 = \delta\}$

which is contractible in T^*N and satisfies

$$\dot{x} = X_H(x)$$

where the Hamiltonian function $H\colon T^*N \to \mathbb{R}$ is given by

$$H(p,q) = \frac{1}{2}|p|^2$$

and X_H is the Hamiltonian vector field of H with respect to the twisted symplectic complex structure

$$\omega_\alpha = -d\lambda - d\pi^*\alpha$$

on T^*N. Here λ denoted the canonical one-form on the cotangent bundle and $\pi\colon T^*N \to N$ the canonical projection.

References

[Fr1] U.Frauenfelder, *Floer homology of symplectic quotients and the Arnold-Givental conjecture*, thesis, (2003).

[Fr2] U.Frauenfelder, *The Arnold-Givental conjecture and moment Floer homology*, math.SG/0309373.

[FrSc] U.Frauenfelder, F.Schlenk, *Hamiltonian dynamics on convex symplectic manifolds*, math.SG/0303282.

Major presentations

November 2001: Floer homology of symplectic quotients, ETH Zürich.

May 2002: Floer homology and Dehn twists, Tel Aviv University.

May 2003: Twisted cotangent bundles and periodic orbits, Hokkaido University.

September 2003: Twisted cotangent bundles and periodic orbits, Okayama University.

November 2003: Moment Floer homology and the Arnold-Givental conjecture, Tokyo University.

December 2003: Moment Floer homology and the Arnold-Givental conjecture, Kyoto University.