POST-DOCTORAL RESEARCH REPORT

Name:	Pho Duc Tai
Host Researcher:	Prof. Ichiro Shimada
Fellowship period:	From November 26, 2002 to November 25, 2004
Title of Research:	Classification and Fundamental group of singular plane curves.

1. RESEARCH DESCRIPTION

1.1. **During 2000-2002.**

1.1.1.   In that period, together with Prof. Mutsuo Oka we published a series of three papers concerning the classification of sextic curves of the (2,3)-torus type, and the fundamental groups of the complement of those curves in $\mathbb{CP}2$.

1.1.2.   I did research on symbolic computations for curves and surfaces, especially with singularities, in fact, I wrote a Maple package SCURVE which is used for compute invariants of singular curves and surfaces.

1.2. **The academic year 2003.**

1.2.1.   The well-known sextic curve with 9 A_2's singularities was introduced by O. Zariski in the late 1928. This curve actually is the dual of the smooth cubic. He also showed that its defining equation $f = 0$ can be decomposed in the sum $f = f_23 + f_32$, where f_2, f_3 are polynomials of degree 2 and 3 respectively. In Kulikov's terminology, this is a (2,3)-torus type. Zariski also calculated the fundamental group of the complement of this curve.

Our motivation started from this. We try to study the dual of smooth quartics and compute the fundamental groups. The dual of a smooth quartic is a curve of degree 12 and of genus 3. It is very helpful for us that there are a rich source of researchs on smooth quartics, such that the research on Fermat curve, Klein curve, etc... We started to check firstly these well-known curves and it is turned out to be again (2,3)-torus type, by the decomposition $f = f_62 + f_43$, where f_6, f_4 are polynomials of degree 6 and 4 respectively.

Exciting with this phenomenon, with Prof. M. Oka and Prof. I. Shimada, we try to prove that this also true for any smooth quartic curves.

Recently, in Jan 2004, we proved the above conjecture. Our next step is to study the fundamental groups of the complement of these dual curves.

1.2.2. During the first year staying at Hokkaido university, I also cooperate with Prof. I. Shimada on the problem of moduli of sextics by the lattice theory which is introduced by V.V.Nikulin.

Beside of these, I am recently cooperate with Dr. Ilias Kotsireas in Waterloo (Canada) on a project of symbolic computation of curves and surfaces. During a week visiting Waterloo (14-21, Feb 2004) we have finished a work on *Intersection numbers of Algebraic Curves*.

2. List of papers

(1) G.Jiang, M.Oka, D.T.Pho and D.Siersma, Lines on Brieskorn-Pham surfaces, *Kodai Mathematical Journal*, Vol. 23, No.2, 214-223, 2000.
(2) D.T.Pho, Note on classification of sextics, *Surikaisekikenkyusho Kokyuroku*, No.1182, 25-42, 2001.
(3) D.T.Pho, Classification of singularities on torus curves of type (2,3), *Kodai Mathematical Journal*, Vol. 24, No.2, 259-284, 2001.
(4) M.Oka and D.T.Pho, Fundamental group of sextics of torus type, in *Trends in Singularities*, ed. by A. Libgober and M. Tibar, Birkhäuser, 151-180, 2002.
(5) M.Oka and D.T.Pho, Classification of sextics of torus type, *Tokyo Journal of Mathematics*, Vol. 25, No. 2, 399-433, 2002.

Others:
(6) S.Kaplan, H.Maakestad, D.T.Pho and M.Teicher, Alexander polynomials and Zariski pairs of sextics (in preparation).
(7) I.S.Kotsireas and D.T.Pho, Intersection numbers of algebraic curves (in preparation).
(8) M.Oka, D.T.Pho and I.Shimada, Dual of smooth quartics (in preparation).

3. List of major talks

(1) On constructive algebraic curves theory, *The third conference on Computer Algebra in Japan*, Tokyo 19 Oct 1999.
(2) On classification of sextics, *Fundamental Groups and Algebraic Functions at RIMS*, Kyoto 7 Jun 2000.
(3) Geometry of sextics of torus type, *Real & Complex Geometry seminar at Tel Aviv university*, 23 Jan 2002.

(4) Alexander polynomials and Zariski pairs of sextics, *workshop on Fundamental Groups in Geometry at MFO (Oberwolfach)*, 08 Sep 2002.
(5) Moduli of certain sextics, Singularity seminar at Tokyo Metropolitan Univ., 14 Apr 2003.

Sapporo, Feb 27, 2004.

Pho Duc Tai