Research Report

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1 During the thesis (University of Marseille - France (09-1999–10-2002))

1.1 Spectral Theory for operators related to the elasticity system.

1.1.1 Topographical waveguide.

This part is devoted to the study of the topographical waveguide for which we proved that there is no guided waves with velocity greater than the one of the S-waves. The question of existence or absence of guided waves propagating with velocities between the one's of the Rayleigh-wave and the S-wave still open. Details are in the paper [2].

1.1.2 Absence of positive eigenvalues for stratified and perturbed elastic system.

In this part we considered stratified and perturbed elastic mediums and we proved results on absence of positive eigenvalues. We considered two types of domains. The first is given by the whole space for which we adapted an idea of W. Littman and R. Weder based on the use of the limiting absorption principle. The second domain is given by a perturbed half-space. For this case, we used the analytic theory of linear operators. Details are in the paper [4].

1.2 One dimensional inverse spectral problem.

We studied the Borg-Levinson problem for which we proved some uniqueness results for discontinuous coefficients. The equation we considered is $-(pu')' + qu = \lambda \rho u$. The new result of this part with respect to the literature is to have taken into account both the three coefficients, p, q and ρ, but we considered piecewise analytic regularities. Details can be found in [3].

2 After the thesis (JSPS fellowship, Hokkaido University-Japan(05-2003—)

2.1 Inverse spectral problems

In the preprint [5], we generalized the results of the section 1.2 to the case where the coefficients p and ρ are Piecewise Lipschitz and q is bounded. We also considered the inverse problem from the nodes.

2.2 Inverse Dynamical problem for connected beams.

We consider an inverse dynamical problem for a composite beam formed by two connected beams. The vibrations of the composite beam are governed by a differential system where a coupling takes place between longitudinal and bending motions. As a first step, we neglect bending motions and we only deal with the longitudinal motions. These motions are governed by a two-by-two second order system coupled in the lower order terms by the shearing stiffness coefficient, which models the connection between the two beams. We give a way of reconstruction of this coefficient from the dynamical Neumann to Dirichlet operator. As a second step, we will take into account the bending motions and try to recover this shearing stiffness coefficient. This is a joint work with Prof. G. Nakamura and Prof. A. Morassi. This is given in [7].

2.3 Inverse problem for waveguide.

In a previous work, Ikehata, Makrakis and Nakamura proposed to use a near field measurement instead of the far field measurement to deal with oceans acoustics. In [5], we clarified the so-called eigenvalue assumption which is one of the obstacles considering this problem. In a work in progress, with Prof. Nakamura we are looking for a way how to state this inverse waveguide problem avoiding this eigenvalue assumption. The aim is also to consider the elastic system.

2.4 Unique continuation for hyperbolic systems.

The aim of this part is to give a unique continuation property for general hyperbolic systems including the general anisotropic elasticity system and to apply it to the continuation of the corresponding Dirichlet to Neuman maps. This result is useful for the inverse dynamical problem. This is a joint work with Prof. G. Nakamura and C-L. Lin.

2.5 Localizing discontinuities of coefficients from finitely and infinitely many measurements.

In this paper, we use the no-response test idea, to identify the interface of the discontinuity ∂D of the coefficient γ of the equation $\nabla \cdot \gamma(x)\nabla + c(x)$ with piecewise regular γ and *bounded* function $c(x)$. We use infinitely many Cauchy data as measurement and give a reconstructive method to localize the interface. We will base this multiwave version of the no-response test on two different proofs. The first one contains a pointwise estimate as used by the singular sources method. The second one is built on an energy (or an integral) estimate which is the basis of the probe method. As a conclusion of this, the probe and the singular sources methods are equivalent regarding their convergence and the no response unifies them.

As a further contribution, we provide a formula to reconstruct the values of the jump of $\gamma(x)$, $x \in \partial D$ at the boundary. This is given in [8].

3 Presentations since 2003.

1) "Absence of positive eigenvalues for stratified and perturbed elastic system. The half-space case."
Seminar at the department of Mathematics of Hokkaido University, June 2003.

2) "Some uniqueness results of discontinuous coefficients for the one dimensional inverse spectral problem."
Seminar at the department of Mathematics of Tokyo Metropolitan University. July 2003.

3)"On the one dimensional Gelfand and Borg-Levinson spectral problems for discontinuous coefficients".
Workshop at Fudan University, Shanghai, $28-30$ **November** 2003.

4)"Inverse dynamical problem for connected beams".
Conference at Tokyo University, $19-21$ **January** 2004

5) " Localizing discontinuities of the conductivity from boundary measurements ".
Seminar in Department of geo-resources, Udine University. February 21st, 2005. Italy

6) "The convergence proof of the no-response test for localizing an inclusion".
The 5th East Asia PDE Conference). January 30st-February 2nd, 2005. Osaka University, Japan.

7) "An inverse dynamical problem for connected beams. The coupled case".
Pde-seminar, department of math, Hokkaido University. November 8 2004. Sapporo, Japan.

8) "The convergence proof of the no-response test for localizing an inclusion".
Taiwan-Japan joint workshop on inverse problems. October 30-November 1, 2004. Taipei, Taiwan.

9)"An inverse dynamical problem for connected beams".
International conference. Modelling and Simulation. June 07-june 12, Fethye, Turkey.

4 Papers.

1) M. Sini: "Un resultat d'absence de valeurs propres plongees pour les guides d'ondes elastiques". CRAS, Serie IIb (2000); $328 : 561 - 564$.

2) M. Sini: "About embedded eigenvalues for a spectral problem arising in the study of surface waves in a topographical waveguide". Math. Math.Appl.Sci. (2002); $25 : 981 - 995$.

3) M. Sini: "Some uniqueness results of discontinuous coefficients for the one dimensional inverse spectral problem." Inverse Problems $19(2003)871 - 894$.

4) M. Sini: "*Absence of positive eigenvalues for linearized elastic systems*". Int. Equ. Oper. Theo. 2004; **49** (2): 255-277.

5) M. Sini: "*On the one dimensional Gelfand and Borg-Levinson spectral problems for discontinuous coefficients.*" Inverse Problems 2004; **20**: 1371-1386.

6) G. Nakamura and M. Sini: "*Near Field Measurements for the Inverse Scattering Problem for the Ocean Acoustics.*" Inverse Problems 2004; **20**: 1387-1392.

7) A. Morassi, G. Nakamura and M. Sini: "*Inverse dynamical problem for connected beams*" to appear in European Journal of Applied Mathematics..

8) G. Nakamura, R. Potthast and M. Sini: "*The convergence proof of the no-response test for localizing an inclusion*". Submitted to Journal of Differential Equations.

9) G. Nakamura, R. Potthast and M. Sini: " *On the multi-wave no response test and its relation with other sampling methods*". To be submitted.