Research Report

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1 During the thesis (University of Marseille - France (09-1999–10-2002))

1.1 Spectral Theory for operators related to the elasticity system.

1.1.1 Topographical waveguide.

This part is devoted to the study of the topographical waveguide for which we proved that there is no guided waves with velocity greater than the one of the S-waves. The question of existence or absence of guided waves propagating with velocities between the one's of the Rayleigh-wave and the S-wave still open. Details are in the paper [2].

1.1.2 Absence of positive eigenvalues for stratified and perturbed elastic system.

In this part we considered stratified and perturbed elastic mediums and we proved results on absence of positive eigenvalues. We considered two types of domains. The first is given by the whole space for which we adapted an idea of W. Littman and R. Weder based on the use of the limiting absorption principle. The second domain is given by a perturbed half-space. For this case, we used the analytic theory of linear operators. Details are in the paper [4].

1.2 One dimensional inverse spectral problem.

We studied the Borg-Levinson problem for which we proved some uniqueness results for discontinuous coefficients. The equation we considered is $-(pu')' + qu = \lambda\rho u$. The new result of this part with respect to the literature is to have taken into account both the three coefficients, p, q and ρ, but we considered piecewise analytic regularities. Details can be found in [3].

2 After the thesis (JSPS fellowship, Hokkaido University-Japan(05-2003—)

2.1 Inverse spectral problems

In the preprint [5], we generalized the results of the section 1.2 to the case where the coefficients p and ρ are Piecewise Lipschitz and q is bounded. We also considered the inverse problem from the nodes.

2.2 Inverse Dynamical problem for connected beams.

We consider an inverse dynamical problem for a composite beam formed by two connected beams. The vibrations of the composite beam are governed by a differential system where a coupling takes place between longitudinal and bending motions. As a first step, we neglect bending motions and we only deal with the longitudinal motions. These motions are governed by a two-by-two second order system coupled in the lower order terms by the shearing stiffness coefficient, which models the connection between the two beams. We give a way of reconstruction of this coefficient from the dynamical Neumann to Dirichlet operator. As a second step, we will take into account the bending motions and try to recover this shearing stiffness coefficient. This is a joint work with Prof. G. Nakamura and Prof. A. Morassi.

2.3 Inverse problem for waveguide.

In a previous work, Ikehata, Makrakis and Nakamura proposed to use a near field measurement instead of the far field measurement to deal with oceans acoustics. In the preprint [6], we clarified the so-called eigenvalue assumption which is one of the obstacles considering this problem. In a work in progress, with Prof. Nakamura we are looking for a way how to state this inverse waveguide problem avoiding this eigenvalue assumption. The aim is also to consider the elastic system.

2.4 Unique continuation for hyperbolic systems.

The aim of this part is to give a unique continuation property for general hyperbolic systems including the general anisotropic elasticity system and to apply it to the continuation of the corresponding Dirichlet to Neuman maps. This result is useful for the inverse dynamical problem. This is a joint work with Prof. G. Nakamura and C-L. Lin.

2.5 Elastography.

Consider an isotropic elastic body. A time harmonic excitation made on the boundary creates a time harmonic wave inside the body. The goal is to give reconstruction algorithms of the elastic material from the knowledge of the amplitude of the wave inside the body. For this purpose, we are starting with a scalar case and consider the localization of discontinuities.
This is a joint work with Prof. G. Nakamura, Prof. K. Nakane and Wang.

3 Presentations.

1) "Absence of positive eigenvalues for stratified and perturbed elastic system. The half-space case."
Seminar at the department of Mathematics of Hokkaido University, June 2003.
2) "Some uniqueness results of discontinuous coefficients for the one dimensional inverse spectral problem."
Seminar at the department of Mathematics of Tokyo Metropolitan University. July 2003.
3)"On the one dimensional Gelfand and Borg-Levinson spectral problems for discontinuous coefficients".
Workshop at Fudan University, Shanghai, $28-30$ November 2003.
4)"Inverse dynamical problem for connected beams".
Conference at Tokyo University, $19-21$ January 2004.

4 Papers.

1) M. Sini: "Un resultat d'absence de valeurs propres plongees pour les guides d'ondes elastiques". CRAS, Serie IIb (2000); $328:561-564$.
2) M. Sini: "About embedded eigenvalues for a spectral problem arising in the study of surface waves in a topographical waveguide". Math. Math.Appl.Sci. (2002); $25:981-995$.
3) M. Sini: "Some uniqueness results of discontinuous coefficients for the one dimensional inverse spectral problem." Inverse Problems 19(2003)$871-894$.
4)M. Sini: "Absence of positive eigenvalues for the linearized elasticity system."
Preprint of University of Provence (2002). To appear in Integral Equations and Operator Theory.
5) G. Nakamura and M. Sini: "Remarks on the Inverse Scattering Problem for the Ocean Acoustics."
Preprint of Math Department, Hokkaido University, December 2003. Submitted for publication.
6) M. Sini: "On the one dimensional Gelfand and Borg-Levinson spectral problems for discontinuous coefficients."
Preprint of Math Department, Hokkaido University, December 2003. Submitted for publication.