## 1. Description of research

1.1. **Conflict sets.** My thesis was published and subsequently reviewed on MathSciNet. Its subject were so-called conflict sets. I will now explain what they are so that I can state what my results were.

Let $M_i$ $1 \leq i \leq l$ be $l$ compact manifolds of codimension one in a smooth ambient manifold $X$, embedded by $\gamma_i \colon M_i \to X$. Finsler metrics on $TX$ are homogeneous Hamiltonians on $T^*X$. Assume that there are given $H_1, \cdots, H_l$ Hamiltonians on $T^*X$, such that all the Finsler metrics are complete and without conjugate or cut-loci. Then we have $l$ distance functions $d_i$, $1 \leq i \leq l$. Each of these gives phase functions

$$F \colon M_i \times X \to \mathbf{R}$$

on each of the $M_i$ namely $F(x, s_i) = d_i(\gamma_i(s_i), x)$. When for some $x \in \mathbf{R}^n$ there are critical points $s_i^0 \in M_i$ of the functions $f_{i,x}$ all having one and the same critical value $x$ is said to lie on the conflict set of the $M_i$. Thus the conflict set is the set of points $x$ that lie at equal distance from a number of submanifolds.

Conflict sets are a generalization of two concepts: medial axis and Voronoi diagram. These last two are widely used in applications. The medial axis is an invariant of shape. It is the most suitable skeleton of a spatial shape for computer storage and manipulation. When in a Voronoi diagram the point sites are replaced by compact submanifolds one obtains a subset of the conflict set of the submanifolds. Such Voronoi diagrams are used in motion planning for robots that need to avoid obstacles.

Fix the $H_i$. For generic embeddings $\gamma_i$ of the $M_i$ the conflict set is the projection of a conic Lagrangian submanifold of $T^*\mathbf{R}^n$. Moreover if $n - l \leq 4$ then there are up to a local diffeomorphism in $\mathbf{R}^n$ a finite number of normal forms of conflict sets. If $l > 2$ then these normal forms are conic Lagrangian manifolds of higher type. Therefore the theorem tells us what the "nice" dimensions for conflict sets are.

In the proof of the theorem as an added bonus we get a method to construct a conic Lagrangian manifold of higher type, using Lagrange multipliers.

In the thesis also many links are shown to exist with such subjects as billiards, affine symmetry. It was shown that in all these domains the theory of Lagrangian and Legendrian singularities, created by Arnol'd and his school is the method to obtain many new results.

1.2. **1-parameter families of conflict sets.** I obtained a complete list of singularities one can expect in a generic one-parameter family of conflict sets in the planar case. There is, as was to be expected, a relation between this list of singularities and singularities of 2-parameter families of fronts in the plane. In particular, dangerous self-tangencies, are unavoidable in such families of conflict sets. There is a duality between these and curves on a torus : the pre-image of the conflict set. Hyperbolic Morse singularities correspond to dangerous self-tangencies. I explain clearly what are the relations between the list of singularities in this geometric setting and in other related settings, such as symmetry sets.

1.3. **Morse functions on Delaunay triangulations.** I have written a joint paper with my former thesis advisor. The paper deals with applications of Morse theory to computational geometry. Delaunay triangulations of point sets in $\mathbf{R}^n$ are defined by distance functions. The Morse theory of the distance function allows one to obtain more information on the shape of the Delaunay triangulation. Delaunay triangulation are widely used in the applied sciences.

The study we carried out is related to several other subjects as well, such as the combinatorics of polytopes and discrete Morse theory.

This is very much an ongoing research project. A second paper is currently in preparation.

1.4. **On affine conflict sets.** With K. Saji, from Hiroshima university I have started a collaboration. From a common interest in generic differential geometry, we study the geometry of affine conflict sets.

1.5. **On cut-loci.** I have written a program to determine the conjugate loci on 2-dimensional surfaces embedded in $\mathbf{R}^3$. This is a non-trivial task, for which none of the standard programs such as Mathematica or Maple are equipped.

I have continued to study closed geodesics on surfaces in 3-space. There are some preliminary results, which I hope will lead up to a publication.

## 2. Publications and submitted papers

- Dirk Siersma and Martijn van Manen, *The nine morse generic tetrahedra*, math.MG/0410251, Submitted
- Martijn van Manen, *Conflict sets, orthotomics, pedals and billiards as canonical relations*, 2004, Submitted, revised version of math.DG/0209148.
- ———, *On dangerous self-tangencies in families of conflict sets*, 2004, Revised version submitted.
- ———, *Curvature and torsion formulas for conflict sets*, Geometry and topology of caustics—CAUSTICS '02, Banach Center Publ., vol. 62, Polish Acad. Sci., Warsaw, 2004, pp. 209–222.
- ———, *The geometry of conflict sets*, Rijksuniversiteit te Utrecht, Utrecht, 2003, Dissertation, Universiteit Utrecht, Utrecht, 2003. pp. x+100, ISBN 90-393-3416-1, MR 2000001
- Rachel Brouwer, Thijs Brouwer, Cor Hurkens, Martijn van Manen, Carolynne Montijn, Jan Schreuder, and J. F. Williams, *Magma design automation: component placement on chips; the "holey cheese" problem*, Proceedings of the Forty-Second European Study Group with Industry (Amsterdam, 2002) (Amsterdam), CWI Syllabi, vol. 51, Math. Centrum Centrum Wisk. Inform., 2002, pp. 77–90.

## 3. Talks and conference visits

- Around primitive forms, Kyoto RIMS, January 2005 ( no talk )
- Symposium on mathematical aspects of image processing and computer vision 2004, Hokkaido university, November 2004
- Hakodate singularity theory conference. September 2004.
- Singularity seminar Hokkaido university, June 2004.
- Jussieu, séminaire sur les singularités. March 2004.
- Séminar "Mathematical challenges from the life sciences", Oberwolfbach, November 2003 ( no talk ).
- Warwick University, November 2003.
- Stafcolloquium Universiteit Utrecht, June 2003.
- Caustics 2002, May 2002, Banach Center, Warsaw.
- Arnol'd Seminar, September 2001, Lomonosov University, Moscow.
- Singularity theory and its applications to geometry, December 2000, University of Liverpool.
- Singularity theory and its applications to wave propagation and dynamical systems, September 2000, Newton Institute Cambridge.