Research report

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My primary area of research is concerned with variational PDE based image restoration, image segmentation and curve evolution methods with applications in medical imaging. Image segmentation is a basic task in image processing. However, there is no unique definition. That is to say that the notion depends on the images that one is dealing with. We take the definition of segmentation to be partitioning of an image f into homogeneous regions and their boundaries such that the regions correspond to objects in the scene. One of the closely related problems to image segmenataion is boundary detection using curve evolution. One of the most celebrated variational PDE based image segmentation model is the Mumford Shah model. Given $f:\Omega\to\mathbb{R}$, a gray scale image, find an optimal piecewise smooth approximation u to f and a set of boundaries K such that u varies smoothly within the connected component of $\Omega\setminus K$ and rapidly across K.

$$\inf\ F_{MS}(u,K)=\int_{\Omega\setminus K}|\nabla u|^2dx+\lambda\int_{\Omega}(u-f)^2dx+\mu Length(K),$$

where K is edge or discontinuity set, Ω is bounded domain in $\mathbb{R}^n$, and λ and μ are positive parameters. It turns out that this is a rather difficult problem to solve because it involves two unknowns: the unkonwn intensity function and unknown set K of edges. A simplified problem of the original Mumford Shah is so called "the minimal partiaion problem." It looks for piecewise constant approximation to f. That is to say, given a function f in $L^\infty(\Omega)$, find a set of disjoint open regions Ω_i, such that $u=c_i$ in each Ω_i is a minimizer of

$$\inf\ F(u,\Gamma)=\sum_i\int_{\Omega_i}|f-c_i|^2dx+\mu\mathcal{H}^{n-1}(\Gamma),$$

where $\Gamma=\cup\partial\Omega_i$, $\Omega=\cup\Omega_i\cup\Gamma$, $\mu>0$ is a scale parameter, and $\mathcal{H}^{n-1}$ is the Hausdorff $(n-1)$ dimensional measure in $\mathbb{R}^n$. Chan and Vese proposed active contour without edge using level set method for representing unknown discontinuity set. The idea is to consider the intensity values inside of contour and not only at their boundaries. The main advantages of their model is they can detect contours with or without gradient, and it automatically detect interior contours. During my Ph.D. study we proposed a more efficient technique for representing unknow discontinuity set via multilayer level set approach. In our approach, we extended their model by using not only the zero level set but other level sets as well to represent boundaries of the objects. The idea is inspired from a different application of implicit curve evolution and free boundaries, introduced in Caflisch et al., where island dynamics for epitaxial growth is modeled. A first layer of islands is represented by $\{x : \phi(x) = 0\}$, then a second layer of islands, growing on the top of the previous one is represented as $\{x : \phi(x) = 1\}$, etc. This new curve evolution approach results in reducing the number of level set functions used and the memory storage required. The energy functional for segmenting 3 phases using one level set function with 0 level set and l level set is given by

$$\inf_{c_1,c_2,c_3,\phi} F(c_1,c_2,c_3,\phi) = \int_\Omega |f(x)-c_1|^2 H(-\phi(x))dx + \int_\Omega |f(x)-c_2|^2 H(\phi(x))H(l-\phi(x))dx + \int_\Omega |f(x)-c_3|^2 H(\phi(x)-l)dx + \mu\Big[\int_\Omega |\nabla H(\phi)| + \int_\Omega |\nabla H(\phi-l)|\Big],$$

One potential application of the multilayer image segmentation is MR brain images, where the task is to partition 3D MR volumetric image into three regions; gray matter (GM), white matter (WM) and cerebro spinal fluid (CSF). In our proposed approach, we first exploit the fact that the non-brain region has intensity value 0. We introduce the characteristic function $\chi_B(x)$ in three dimensions for the brain region. That is $\chi_B(x) = 1$ when x belongs to the brain region and $\chi_B(x) = 0$ when x belongs to the non-brain region. We then evolve the surface only within the region where $\chi_B(x) = 1$, by restricting all energy terms to the brain region. This avoids segmenting the non-brain region and thus reduces non-desired artifacts and complexity. One function with two levels is sufficient to segment the volumetric MR images into three tissue types. We propose to minimize the energy

$$\begin{aligned} F(c_{GM},c_{WM},c_{CSF},\phi) &= \int_\Omega |f-c_{GM}|^2 \chi_B(x) H(-\phi(x))dx + \int_\Omega |f-c_{WM}|^2 \chi_B(x) H(\phi(x))H(l-\phi(x))dx \\ &+ \int_\Omega |f-c_{CSF}|^2 \chi_B(x) H(\phi(x)-l)dx + \mu\Big[\int_\Omega |\nabla H(\phi)|\chi_B(x)dx \\ &+ \int_\Omega |\nabla H(\phi-l)|\chi_B(x)dx\Big]. \end{aligned}$$

In the academic year of 2007, we proposed a region based active contour model for image segmentation and restoration in a variational level set framework which takes into account blur. This model is based on the assumption that the observed image is roughly piecewise-constant with blurring operator being applied on it. The main motivation of our proposed model arises from medical imaging such as MR brain images which exhibit intensity inhomogeneity and blurry boundaries. Since I was employed by the Department of Mathematics at Hokkido University, I have been involved with a project which investigates the structural atypia of cancer by analyzing unsupervised segmentation of tissue histology images.

Journal publications

- G. Chung AND L. Vese,

  Image Segmentation Using a Multilayer Level-Set Approach, Computing and Visualization in Science 2007, accepted.

Refereed conference publications

- G. Chung, T. Le, L. Lieu, N. Tanushev, AND L. Vese

  Computational Methods for Image Restoration, Image Segmentation, and Texture Modeling, SPIE Electronic Imaging 2006.

- G. Chung AND L. Vese,

  Energy Minimization Based Segmentation and Denoising Using Multilayer Level Set Approach, EMMCVPR 2005, LNCS Vol. 3757/2005, pp 439-455.

Preprints

- G. Chung,

  Variational Image Segmentation and Restoration Using a Multilayer Imaplicit Curve Evolution Method, Ph.D. thesis, May 2007.

- C. Bailey, G. Chung, A. Guevara, S. Hardesty, J. Kenney, S. Sircar AND D. Allan,

  Birefringence Data Analysis, IMA Preprint Series ♯ 2133-1, 1-19, September 2006.

Presentations

- COE Research Course, *Introduction to Variational Models in Image Processing,* October 9 - December 20, 2007, Hokkaido University, Japan.

- Mathematical Aspects of Image Processing and Computer Vision (MAIPCV), *Variational image segmentation using multilayer implicit curve evolution approach,* November 22, 2007, Sapporo, Japan

- SIAM conference on Computational Science and Engineering, computational aspects of medical imaging, *Volumetric MRI brain segmentation using a multilayer surface evolution approach,* February 21, 2007, Costa Mesa, CA, U.S.A.

- Special session on nonsmooth analysis in inverse and variational problems at the AMS Joint Mathematical Meeting, *Multilayer Segmentation and Application to MR Brain imaging,* January 7, 2007, New Orleans, LA, U.S.A.

- Computational advances in evolving curves and surfaces at the 7th World Congress on Computational Mechanics, *Motion of Curves and Surfaces for Multilayer Image Segmentation,* July 17, 2006, Century City, CA, U.S.A.

- SIAM Imaging Science Mini-Symposium, *Image Segmentation Using a Multilayer Level Set Approach with Application to MRI data,* May 16, 2006, Minneapolis, MN, U.S.A.

- International Workshop on Energy Minimization Methods in Computer Vision and Pattern Recognition, *Energy Minimization Based Segmentation and Denoising Using a Multilayer Level Set Approach,* November 9-11, 2005, St. Augustin, FL, U.S.A.