RESEARCH REPORT

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1. Research Activities

I have been working on several issues on Partial Differential Equations (PDEs) in 2006. I studied the semirelativistic Hartree equations and (in)compressible Navier-Stokes equations for the fluids with vacuum.

2. Research background

1. Semirelativistic Hartree equations: The physical models are originated from the Boson stars which are quantum mechanical system of massive or massless Boson particles with Coulomb or gravitational interaction defined by the convolution with $|x|^{-\gamma}$ for some positive number $\gamma$ less than the space dimension $n$. The equation is to be derived by the method of mean field dynamics. The main problem concerning this equation is to establish the global well-posedness of solutions and their scattering according to the value of $\gamma$.

2. Navier-Stokes equations: The global existence of weak solution of Navier-Stokes equation with vacuum is well-known but the uniqueness remains still open even for two dimensional case. It has been known that to guarantee the uniqueness, a strong regularity is inevitable. But there has almost not been known for a general fluid with vacuum. In general, the regularity of solution is mainly gained by the parabolicity of the momentum equation. But in the presence of the vacuum, the momentum equation looses the parabolicity. Thus we need a compatibility condition which turns out to be necessary and sufficient for a solution to have the strong regularity. Here the problem is how to get a high regularity of solution and extend to more complicated models.

3. Research Methods

1. Semirelativistic Hartree equations: The main tool is the Strichartz estimate associated with Klein-Gordon equation. We also need a time decay of linear solution for which we use the finite propagation speed of solutions.

2. Navier-Stokes equations: We get a uniform bound of linear solutions is necessary to use the usual iteration scheme. We first control the density of solution and then the velocity( temperature and their high regularity) via a bootstrap argument.

4. Results

1. Semirelativistic Hartree equations: In the papers [5,12], the global time behavior was shown by a careful observation of the conservation laws and Strichartz estimates. In particular the radially symmetric solutions were considered in [12]. In [13] we considered a potential which is the sum of two potentials with different decay rates. One of the most interesting thing is the behavior of solutions when the mass is large (we call this nonrelativistic limit). In [5] we showed the solution
with large mass of semirelativistic equation behaves like a solution of Schrödinger equation with the same nonlinearity.

2. Navier-Stokes equations In [4], we discussed a polytropic compressible fluid with vacuum. We applied the method in [4] to the regularity problem of the barotropic compressible Navier-Stokes equations [11] and heat conducting incompressible Navier-Stokes equations [9]. The most important results is the local strong solvability which is sharp in view of the recent result [2] where we proved that there is no global strong solution if the initial density has compact support.

5. Papers from 2006

6. Talks

[1] Unique solvability of the initial boundary value problems for compressible viscous fluids, International Conference on Nonlinear PDE and Related Topics: Celebrating Neil Trudinger’s 60th Birthday, Australian National University, Australia, 2002.07
[2] Local existence for viscous polytropic fluid with vacuum, The 1st PDE Workshop of Educational Science Institute, Cheju National University, Korea, 2003.04
[3] Local existence for heat-conducting incompressible fluid, The 6th Workshop on Differential Equations, Mathematical Research Center, Chonnam National University, Korea, 2003.08
[6] Sharp boundedness of Bochner-Riesz operator with negative index, PDE Seminar, Hokkaido University, Japan, 2005.03
[7] Lorentz space extension of Strichartz estimates, Wave Seminar, Hokkaido University, Japan, 2005.04
[8] On the Bochner-Riesz operator with negative index, Harmonic Analysis and Partial Differential Equation, University of Kiel, Germany, 2005.06
[9] On classical solutions of the compressible Navier-Stokes equation with nonnegative density, The 15th PDE Real Analysis Seminar, University of Tokyo, Japan, 2005.07
[13] Well-posedness of semi-relativistic Hartree type equation, Wave Seminar, Hokkaido University, Japan, 2006. 05