RESEARCH REPORT

Yonggeun Cho, Post Doctoral Fellow
Department of Mathematics, Hokkaido University

1. Research Activities

My research concerns in 2005 are on several problems issued in Partial Differential Equations (PDEs) and Harmonic Analysis. I studied the dispersive equation (mainly generalized Boussinesq equation) and Navier-Stokes equations for the fluids with vacuum for PDE part, and also a maximal inequality associated to the Schrödinger type equation for Harmonic Analysis.

1.1. PDEs.

1.1.1. *Generalized Boussinesq equations.* Prof. Ozawa and I have tried to prove the global existence of small amplitude solutions and scattering of generalized Boussinesq equations which governs the physical models like shallow water waves, ion-sound waves, longitudinal variation of elastic rod, FPU problem and so on [2, 18]). The waves (solutions) evolve as the result of competition of dispersion and nonlinearity. Here the nonlinearity is described usually as a power type function like u^p. One of the most important problems is to find the smallest power p for the global existence and scattering. In the paper [9], we improved the previously known result ($p > 8$ [17, 25, 26]) up to $p > \frac{9}{2}$. Furthermore, we also proved non-existence of asymptotically free solutions for $1 < p \leq 2$.

We considered a system of Schrödinger and IMBq equations which describes a model of ion-sound waves with velocity near Mach number. In [10], we proved the global existence and scattering for the dimension 1 and 2. Moreover, we provided a blow-up criterion for 3 and 4 dimensional cases.

1.1.2. *Navier-Stokes equations.* I focused on the local existence and uniqueness of strong solution deriving the evolution of fluids with vacuum. The global existence of weak solution of Navier-Stokes equation with vacuum is well-known but the uniqueness remains still open even for two dimensional case. It has been known that to guarantee the uniqueness, a strong regularity is inevitable. But there has almost not been known for a general fluid with vacuum. In general, the regularity of solution is mainly gained by the parabolicity of the momentum equation. But in the presence of the vacuum, the momentum equation looses the parabolicity. Thus we need a compatibility condition which turns out to be necessary and sufficient for a solution to have the strong regularity.

In [5], Prof. Kim and I discussed a polytropic compressible fluid with vacuum. The most difficult one causing obstacles against strong regularity is the quadratic nonlinearity $|\mathrm{d}u|^2$ in the energy equation which correlates heavily ρ, u and e. Moreover, the vacuum deepens the correlation. To overcome this complexity, we examined elaborately the correlation and obtained a strong estimate which can be applied to the regularity problem of the barotropic compressible Navier-Stokes equations [6] and heat conducting incompressible Navier-Stokes equations [7]. The most important results is the local strong solvability which is sharp in view of the recent result [4] where we proved that there is no global strong solution if the initial density has compact support.

1.2. **Harmonic Analysis.** The problem concerned is to prove that the solution $u(x,t) = e^{it(-\Delta)^{\frac{a}{2}}} f(x)$ of linear dispersive equation $iu_t + (-\Delta)^{\frac{a}{2}} u = 0, u(0) = f$ has the mapping properties that $\| \sup_{t\in\mathbb{R}} |u(\cdot,t)| \|_{L^p} \lesssim \|f\|_{H^{\frac{1}{4}}}$ for any $a > 0 (\neq 1)$ and for some $p \geq 2$.

This problem was initiated by L. Carleson for $a = 2$ [3] and extensively studied by many people, for instance see [1, 3, 11–16, 19–24]. But it remains still open. Prof. Shim, Prof. Lee and I could prove partially that this is true for functions of finite linear combination of radial and spherical harmonic functions by using asymptotic behavior of Bessel functions and a boundedness of one dimensional oscillatory integrals [8, 12].

References

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2. List of Papers

2.1. Published papers.

1. (with E. Koh and S. Lee) *A maximal inequality for filtration on some function spaces*, Osaka J. Math. **41** (2004), 267-276.
2. (with H. J. Choe and H. Kim) *Unique solvability of the initial boundary value problems for compressible viscous fluids*, J. Math. Pures Appl. **83** (2004), 243-275.
3. (with H. Kim) *Unique solvability for the density-dependent Navier-Stokes equations*, Nonlinear Analysis **59** (2004), 465-489.
4. (with Y. Shim) *Weighted L^2 estimates for maximal operators associated to dispersive equations*, Illinois J. Math. **48** (2004), 1081-1092.
5. (with Y. Kim, S. Lee and Y. Shim) *Sharp $L^p - L^q$ estimates for Bochner-Riesz operators of negative index in $\mathbb{R}^n, n \geq 3$*, J. Func. Anal. **218** (2005) 150-167.
6. (with C. Ahn) *Lorentz space extension of Strichartz estimates*, Proc. A.M.S. **133** (2005) 3497-3503.

2.2. Accepted papers.

1. (with B.J. Jim) *Blow-up of the viscous heat-conducting compressible flow*, to appear in J. Math. Anal. Appl..
2. (with S. Lee and Y. Shim) *A maximal inequality associated to Schorödinger type equation*, to appear in Hokkaido Math. J..
3. (with T. Ozawa) *Remarks on modified improved Boussinesq equations in one space,* to appear in Proc. Roy. Soc. Lond. A.
4. (with H. Kim) *On the classical solutions of the compressible Navier-Stokes equations with nonnegative initial densities*, to appear in manuscripta mathematica.

2.3. Preprints.

1. (with H. Kim) *Existence results for viscous polytropic fluids with vacuum*, Hokkaido Univ. Preprint Series in Math. ♯675, 2004.

2. (with Y. Shim) *Global estimates of maximal operators generated by dispersive equations*, Hokkaido Univ. Preprint Series in Math. ♯704, 2005.
3. (with H. Kim) *Existence result for heat-conducting viscous incompressible fluids with vacuum,* Hokkaido Univ. Preprint Series in Math. ♯742, 2005.
4. (with T. Ozawa) *Global existence on nonlinear Schrödinger-IMBq equations,* Hokkaido Univ. Preprint Series in Math. ♯744, 2005.
5. (with T. Ozawa) *On small amplitude solutions to the generalized Boussinesq equations,* Hokkaido Univ. Preprint Series in Math. ♯764, 2006.
6. (with T. Ozawa) *On the semi-relativistic Hartree type equation,* Hokkaido Univ. Preprint Series in Math. ♯773, 2006.
7. *High regularity of solutions of compressible Navier-Stokes equations,* Hokkaido Univ. Preprint Series in Math. ♯776, 2006.

3. **List of Presentations**

1. *Asymptotic behavior of Nonlinear Schrödinger equation*, Academic Seminar, KIAS, Korea, 2002.
2. *Unique solvability of the initial boundary value problems for compressible viscous fluids,* International Conference on Nonlinear PDE and Related Topics: Celebrating Neil Trudinger's 60th Birthday, Australian National University, Australia, 2002.07
3. *Local existence for viscous polytropic fluid with vacuum,* The 1st PDE Workshop of Educational Science Institute, Cheju National University, Korea, 2003.04
4. *Local existence for heat-conducting incompressible fluid,* The 6th Workshop on Differential Equations, Mathematical Research Center, Chonnam National University, Korea, 2003.08
5. *Regularity results for viscous compressible fluids with vacuum,* The 12th Applied Mathematical Forum, SoAnbo, Korea, 2004.02
6. *Blow-up of the viscous heat-conducting compressible flow,* The 6th Northeastern Symposium on Mathematical Analysis, Tohoku University, Japan, 2005.02
7. *Sharp boundedness of Bochner-Riesz operator with negative index,* PDE Seminar, Hokkaido University, Japan, 2005.03
8. *Lorentz space extension of Strichartz estimates,* Wave Seminar, Hokkaido University, Japan, 2005.04
9. *On the Bochner-Riesz operator with negative index,* Harmonic Analysis and Partial Differential Equation, University of Kiel, Germany, 2005.06
10. *On classical solutions of the compressible Navier-Stokes equation with nonnegative density,* The 15th PDE Real Analysis Seminar, University of Tokyo, Japan, 2005.07
11. *An improvement of Bochner-Riesz problem,* Mathematical Seminar, KIAS, Korea, 2005.07
12. *Boundedness of Fourier multiplier operator defined by elliptic type function,* Harmonic Analysis and its Application, Hokkaido University, Japan, 2005.08
13. *Long time behavior of small amplitude solutions of generalized Boussinesq and modifed improved Boussinesq equations,* Nonlinear Wave Equations, Sapporo Guest House Symposium, Japan, 2005.11