Brian Forbes
1)Research report.

The underlying theme of all my work has been to understand the mathematical content of dualities predicted by physics. Specifically, in the area of string theory, physics provides a long list of duality statements between Calabi-Yau manifolds. The manifolds on both sides of the duality may be compact or noncompact, and may include extra information, such as Lagrangian or holomorphic submanifolds of the respective Calabi-Yaus.

In my dissertation, "Open string mirror maps from Picard- Fuchs equations on relative cohomology", I examined the physically conjectured correspondence between type A and type B open string theory on noncompact toric Calabi-Yau manifolds. Mathematically, this amounts to a local isomorphism of moduli spaces $(\mathcal{X},\mathcal{L}) \cong (\mathcal{Y},\mathcal{C})$. Here $(\mathcal{X},\mathcal{L})$ consists of the complexified Kähler moduli space of a representative $X \in \mathcal{X}$ together with the space of deformations of a Lagrangian submanifold $L \subset X$; $(\mathcal{Y},\mathcal{C})$ is the combination of the complex moduli space of $Y \in \mathcal{Y}$ and the space of deformations of a holomorphic curve $C \subset Y$. I established the correspondence by defining a notion of "relative period integrals" for the pair $(\mathcal{Y},\mathcal{C})$, and demonstrating that they were annihilated by a new set of Picard- Fuchs differential operators. I then showed that the Picard- Fuchs operators obtained from my construction agreed with those emerging from physics.

My second project, "Computations on B model geometric transitions", was aimed at exploring the mathematical implications of geometric transitions. The basic conifold transition takes open type A string theory on $X = T^*S^3$ to closed A string theory on $\hat{X} = \mathcal{O}(-1) \oplus \mathcal{O}(-1) \to \mathbb{P}^1$. If we set $L = S^3$, then this implies a moduli space isomorphism $(\mathcal{X},\mathcal{L}) \cong \hat{\mathcal{X}}$. It is possible to consider more general geometric transitions in any noncompact Calabi-Yau $\hat{\mathcal{X}}$ containing a holomorphic curve $C \hookrightarrow \hat{\mathcal{X}}$ such that $\mathcal{N}_{C/\hat{\mathcal{X}}} \cong \mathcal{O}(-1) \oplus \mathcal{O}(-1)$. We simply perform the extremal geometric transition given by contraction of C, followed by a resolution of the resulting singularity via complex deformation. In order to check the physically proposed duality $(\mathcal{X},\mathcal{L}) \cong \hat{\mathcal{X}}$ of these examples, I compared the relevant integrals on each side of the transition and demonstrated equivalence. Moreover, I showed that not all of the data claimed by physics can be shown mathematically. In fact, the conifold transition itself contains no information from a mathematical perspective.

Currently, Masao Jinzenji and I are working on a collaborative project on the subject of local mirror symmetry. Local mirror symmetry is usual mirror symmetry between Calabi-Yaus, except that the manifolds are allowed to be noncompact. In the event that the Calabi-Yau is given by the total space of the canonical bundle of a Fano surface S, Chiang, Klemm, Yau and Zaslow have provided a recipe for the calculation. Moreover, recent work of Hosono has expanded the applicable cases to include those whose Betti numbers satisfy $b_2 = b_4$. Professor Jinzenji and I have devised an extended set of Picard- Fuchs operators (whose solutions contain those found in usual mirror symmetry as a subset) which, in many cases, can be shown to complete the missing mirror symmetry data. We are also able to derive predictions in certain cases where local mirror symmetry fails.

(i) **Publications**.

1) B. Forbes, *Open string mirror maps from Picard- Fuchs equations on relative cohomology,* hep-th/0307167. Submitted to Communications in Mathematical Physics. July, 2003. Published in the Hokkaido University Preprint Series in Mathematics.

2) B. Forbes, *Computations on B model geometric transitions,* hep-th/0408167. Submitted to the Journal of High Energy Physics. August, 2004. Published in the Hokkaido University Preprint Series in Mathematics.

3) M. Jinzenji and B. Forbes, *Extending the Picard-Fuchs system of local mirror symmetry,* in preparation.

(ii) **Lectures**.

1) *Open string mirror maps from Picard- Fuchs equations on relative cohomology,* given at "Calabi-Yau Varieties and Mirror Symmetry", Banff International Research Station. Banff, Alberta, Canada. December, 2003.

2) *Computations on B model geometric transitions,* given at "Mirror Symmetry", sponsored by Kyoto University. Fukui, Japan. October, 2004.

3) *Extending the Picard-Fuchs system of local mirror symmetry,* given at Hokkaido University Physics Department. Sapporo, Japan. November 2004.

4) *Extending the Picard-Fuchs system of local mirror symmetry,* given at Tokyo University. Tokyo, Japan. January 2005.

5) *Extending the Picard-Fuchs system of local mirror symmetry,* given at Tokyo Metropolitan University. Tokyo, Japan. January 2005.