Recent developments in open mirror symmetry

Satoshi Minabe

November 19–21, 2007

Let us start with the following classical problem.

Problem 1. Let Q be a quintic hypersurface in the 4-dimensional complex projective space. How many rational curves of degree d are there on Q ?

Here the numbers of rational curves are defined via Gromov–Witten theory. The following answer to Problem 1 is known. Consider the generating series of the numbers of rational curves (Gromov–Witten invariants) with respect to the degree d. Then it is related to certain hypergeometric series via certain change of variables called the mirror transformation. This fact is called the mirror theorem. It was discovered by Candelas et al. around 1990 based on the mirror symmetry, which is a kind of dualities in topological string theories. The first mathematical proof was given by Givental in 1995.

Let us turn to a real version of the Problem 1.

Problem 2. Let Q be a quintic hypersurface equipped with a real structure. How many real rational curves of degree d are there on Q ?

If d is odd, this is equivalent to the enumeration of holomorphic discs whose boundaries lie on the set $Q(\mathbb{R})$ of real points of Q with respect to the given real structure. In 2006, Walcher conjectured an answer to the problem of disk counting. His conjecture was proved by Pandharipande–Solomon–Walcher in the same year. The result is the following 'open' mirror theorem : the generating series of the numbers of real rational curves is related to a certain hypergeometric series via the mirror transformation. The proof uses a theory of open Gromov–Witten invariants and Givental's mirror theorem. Their result may also be seen as an answer to the Problem 2.

The goal of this series of talks is to give an overview of the work of Pandharipande–Solomon–Walcher. Special attention will be paid to localization calculations in the proof. Basic facts on Gromov–Witten theory will also be explained in due course.